Subjects covered:

Transmission Lines:

$$-\frac{\partial V}{\partial z} = RI + L\frac{\partial I}{\partial t} \qquad \frac{d^2V}{dz^2} - \gamma^2 V = 0$$
$$\Rightarrow$$
$$-\frac{\partial I}{\partial z} = GV + C\frac{\partial V}{\partial t} \qquad \frac{d^2I}{dz^2} - \gamma^2 I = 0$$

2 coupled first order Differential Equations➔uncoupled second order differential equation (wave equation)

γ is the complex propagation constant➔ $\gamma = \alpha + j\beta = \sqrt{(R' + j\omega L')(G' + j\omega C')}$
α is the attenuation constant
β is the phase constant

Defined the Characteristic Line impedance: $Z_0 = \sqrt{\frac{(R' + j\omega L')}{(G + j\omega C')}}$

Defined the Voltage Reflection Coefficient: $\Gamma = \frac{Z_L - Z_0}{Z_L - Z_0}$

The reflected wave interferes with the transmitted wave to form a standing wave in the transmission line. The ratio of the maximum voltage of the standing wave to the minimum is the Standing Wave Ratio:

$SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}$ used to measure the mismatch between load and transmission line.

The standing wave pattern implies there are oscillations in the voltage and current as a function of position. The ratio of voltage to current as a function of position is the input impedance also a function of position:

$$Z_{in}(-l) = Z_0\left[\frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}\right] \qquad \text{lossy line}$$

$$Z_{in}(-l) = Z_0\left[\frac{Z_L + Z_0 \tan \beta l}{Z_0 + Z_L \tan \beta l}\right] \qquad \text{lossless line}$$

Can use the Smith Chart to plot input impedance as a function of position (WTG), or move a connect closer to the load (WTL), SWR, impedance match with single stub, or impedance match with a double stub.

Electromagnetic fields are a solution to the wave equation.

Field direction is perpendicular to the direction of propagation.
Electric and magnetic fields are perpendicular. These two statements are summarized in the expressions:

$$\vec{H} = \frac{1}{\eta}\hat{k} \times \vec{E} \qquad\qquad \vec{E} = -\eta\,\hat{k} \times \vec{H}$$

These also relate the amplitudes of the fields. The field amplitudes are scaled by the impedance of the medium.

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_r \mu_0}{\varepsilon_r \varepsilon_0}} = \sqrt{\frac{\mu_r}{\varepsilon_r}}\eta_0$$

The permittivity can be complex; this implies a frequency sensitive loss component.

$\varepsilon_c = \varepsilon - j\dfrac{\sigma}{\omega} \Rightarrow \eta_c$ This implies a complex impedance and a phase shift between the electric and magnetic fields.

The flow of power carried by an EM field is determined by the Poynting Vector

$\vec{S} = \vec{E} \times \vec{H}$ this oscillates at the wave frequency, more often interested in the average power density: $\vec{S}_{ave} = \dfrac{1}{2}\mathrm{Re}\left\{\vec{E} \times \vec{H}^*\right\} = \dfrac{1}{2\eta}|E|^2 = \dfrac{\eta}{2}|H|^2$

Polarization defines the electric field vector direction as the wave propagates, the most general case is elliptical, with circular and linear being special cases.

EM waves at boundaries:

There is a direct correlation to the fields in the unbounded normal case and the voltage in the bounded case (Transmission lines) The fields will reflect when there is an impedance mismatch in the media.

$$\Gamma = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad\qquad \mathrm{T} = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

When the wave comes into an interface off of normal, the reflected wave leaves at the same angle as incident angle and the transmitted wave is refracted by Snells Law. Tthe polarization becomes important to determine the reflection and transmission coefficients. These are determined by the Fresnel equations:

$$\Gamma_{||} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} \qquad T_{\parallel} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$\Gamma_{\perp} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_{ti}}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} \qquad T_{\perp} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

Waveguides

Fields in a hollow conductor. Act as a filter with a cutoff frequency

The cutoff frequency is a function of the materials and the geometry

$$f_c = f_c(u', a, b) = \frac{u'}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

The waveguide will have an impedance that is function of the materials, geometry, the operating frequency and the mode:

$$\eta_{TE} = \eta_{TE}(\eta', f_c, f) = \frac{\eta'}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \qquad \lim f \to f_c \Rightarrow \eta_{TE} = \infty$$

$$\eta_{TM} = \eta'\sqrt{1 - \left(\frac{f_c}{f}\right)^2} \qquad \lim f \to f_c \Rightarrow \eta_{TM} = 0$$

$$\lim f \to \infty \Rightarrow \eta_{TM} = \eta_{TE} = \eta'$$

Antennae

Normalized radiation pattern: $F(\theta,\phi)$ which directions do the antenna radiated and the relative magnitudes of the pattern.

Radiation Intensity: $U(\theta,\phi)$ time average radiated power per unit solid angle

$$U(\theta,\phi) = R^2|S_{ave}| \qquad P_{rad} = \iint U d\Omega \qquad U_{ave} = \frac{P_{rad}}{4\pi}$$

Directive Gain: $$G_D = \frac{U(\theta,\phi)}{U_{ave}} = \frac{4\pi\, U(\theta,\phi)}{P_{rad}}$$

Directivity: $$D = \frac{U_{\max}}{U_{ave}} = G_D(\max)$$

Radiation efficiency $$\xi = \frac{P_{rad}}{P_{in}}$$

Gain accounts for ohmic losses and directivity does not: $G = \xi D$

Radiation Resistance:

Hypothetical resistor that would dissipate a power equal to the power radiated by the antenna when feed by the same current

Effiency then becomes $$\xi = \frac{R_{rad}}{R_{rad} + R_{Loss}}$$

Beam width: width of beam define from half intensity points $F(\theta,\phi) = .5$ (-3dB) measured in angles

How does a dipole antenna radiate?

A charge creates an electric field, and a current creates a magnetic field. A changing current (accelerates a charge) creates both a changing electric field and a changing magnetic field, which couple and create an electromagnetic wave that is self propagating.